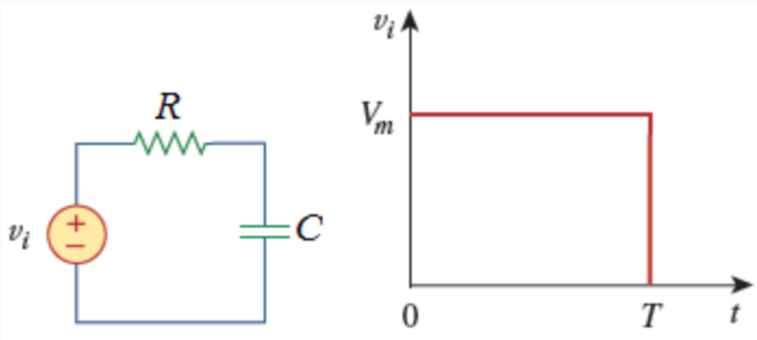
1.

value:
10.00 points

The circuit in figure (a) can be designed as an approximate differentiator or an integrator, depending on whether the output is taken across the resistor or the capacitor, and also on the time constant $\tau = RC$ of the circuit and the width T of the input pulse in figure (b). The circuit is a differentiator if $\tau << T$, say $\tau < 0.1T$, or an integrator if $\tau >> T$, say $\tau > 10T$. Assume $R = 500\ \text{k}\Omega$ and $C = 400$ pF.



(a) (b)

What is the minimum pulse width that will allow a differentiator output to appear across the capacitor?

The minimum pulse width that will allow a differentiator output to appear across the capacitor is *2.0 ± 2%* ms.

Explanation:

$$\tau = RC = \left(400 \times 10^{3}\right)\ \Omega \times \left(400 \times 10^{-12}\right)\ \text{F} = 160.00\ \mu\text{s}$$

As a differentiator,
$$T > 10\tau = 1600.00\ \mu\text{s} = 1.6\ \text{ms}$$

i.e., $T_{\text{min}} = 1.6\ \text{ms}$

The minimum pulse width that will allow a differentiator output to appear across the capacitor is 1.6 ms.

2.

value:
10.00 points

An RL circuit may be used as a differentiator if the output is taken across the inductor and $\tau << T$ (say τ < 0.1 *T*), where *T* is the width of the input pulse. If *R* is fixed at 320 kΩ, determine the maximum value of *L* required to differentiate a pulse with *T* = 10 μs.

The maximum value of *L* required to differentiate a pulse with *T* = 10 μs is < *320 ± 2%* mH.

Explanation:

Since $\tau < 0.1\ T = 1\ \mu\text{s}$.

$\frac{L}{R} < 1\ \mu\text{s}$

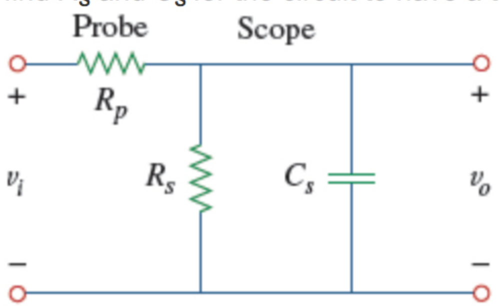
$L < R \times 10^{-6} = (360 \times 10^{3})\ \Omega \times (1 \times 10^{-6})\ \text{s} = 360\ \text{mH}$

The maximum value of *L* required to differentiate a pulse with *T* = 10 μs is < 360 mH.

3.

value:
10.00 points

An attenuator probe employed with oscilloscopes was designed to reduce the magnitude of the input voltage v_i by a factor of 10. As shown in the given figure, the oscilloscope has internal resistance R_s and capacitance C_s, while the probe has an internal resistance R_p. If R_p is fixed at 6 MΩ, find R_s and C_s for the circuit to have a time constant of 24 μs.



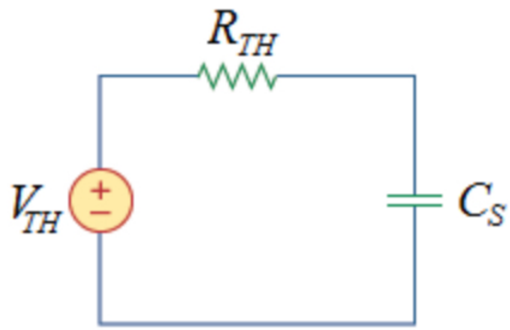
The value of R_s is 0.67 ± 2% MΩ.

The value of C_s is 40.00 ± 2% pF.

Explanation:

We determine the Thévenin equivalent circuit for the capacitor C_S.

$v_{TH} = \frac{R_s}{R_s + R_p} v_i$, $R_{TH} = R_s \| R_p$



The Thévenin equivalent is an RC circuit.

Since $v_{TH} = \frac{1}{10} v_i \rightarrow \frac{1}{10} = \frac{R_s}{R_s + R_p}$,

$R_s = \frac{1}{9} R_p = \frac{6}{9} = \frac{2}{3}\ \mathrm{M\Omega} = 0.667\ \mathrm{M\Omega}$

Also, $\tau = R_{TH} C_s = 19\ \mu s$

$R_{TH} = R_p \| R_s = \frac{6\left(\frac{2}{3}\right)}{6 + \left(\frac{2}{3}\right)} = 0.6\ \mathrm{M\Omega}$

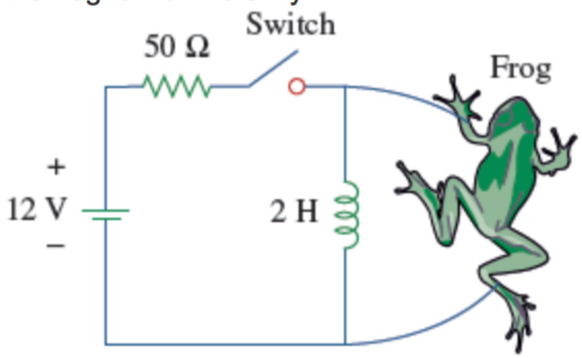
$C_s = \frac{\tau}{R_{TH}} = \frac{\left(19 \times 10^{-6}\right) s}{\left(0.6 \times 10^{6}\right) \Omega} = 31.67\ \mathrm{pF}$

The value of R_S is 0.67 MΩ.
The value of C_S is 31.67 pF.

4.

value:
10.00 points

The circuit in the given figure is used by a biology student to study “frog kick.” She noticed that the frog kicked a little when the switch was closed but kicked violently for 9 s when the switch was opened. Model the frog as a resistor and calculate its resistance. Assume that it takes 10 mA for the frog to kick violently.



The resistor value is 0.706 ± 2% Ω.

Explanation:

The resistor value can be determined as follows:

$$i_o(0) = \frac{12\text{ V}}{50\,\Omega} = 240\text{ mA}$$

$$i(\infty) = 0$$

$$i(t) = i(\infty) + (i(0) - i(\infty))\,e^{-t/\tau}$$

$$i(t) = 240\,e^{-t/\tau}\text{ mA}$$

$$\tau = \frac{L}{R} = \frac{2\text{ H}}{R}$$

$$i(t_o) = 10\text{ mA} = 240\,e^{-t_o/\tau}\text{ mA}$$

$$e^{-t_o/\tau} = 24 \rightarrow t_o = \tau\ln(24)$$

$$\tau = \frac{t_o}{\ln(24)} = \frac{7\text{ s}}{\ln(24)} = 2.203\text{ s} = \frac{2\text{ H}}{R}$$

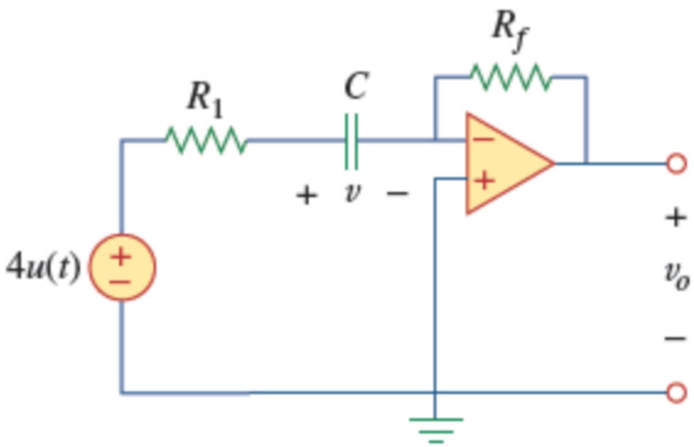
$$R = \frac{2\text{ H}}{2.203\text{ s}} = 0.908\,\Omega$$

The resistor value is 0.908 Ω.

5.

value:
10.00 points

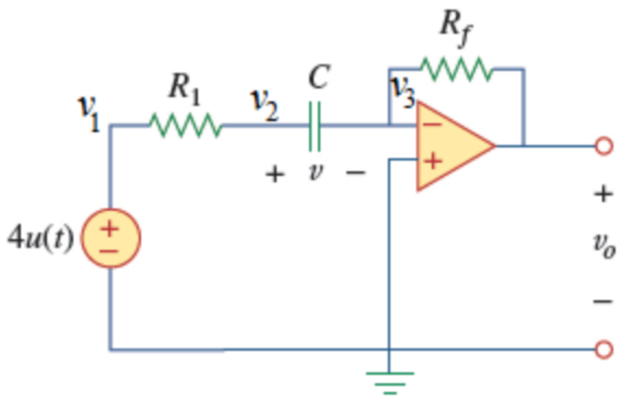
For the op amp circuit of the given figure, let R_1 = 10 kΩ, R_f = 30 kΩ, C = 40 μF, and $v(0)$ = 1 V. Find v_o.



The voltage v_o is ((-9.00 ± 2%)$e^{-t/}$ 0.40 ± 2%)$u(t)$ V.

Explanation:

Consider the circuit below.



At node 2,
$\frac{v_1 - v_2}{R_1} = C\frac{dv}{dt}$ ______ (1)

At node 3,
$C\frac{dv}{dt} = \frac{v_3 - v_o}{R_f}$ ______ (2)

But $v_3 = 0$ and $v = v_2 - v_3 = v_2$.

Hence, (1) becomes
$\frac{v_1 - v}{R_1} = C\frac{dv}{dt}$

$v_1 - v = R_1 C\frac{dv}{dt}$ or $\frac{dv}{dt} + \frac{v}{R_1 C} = \frac{v_1}{R_1 C}$

Hence, $v(t) = \begin{cases} v_T & t = 0^- \\ v_1 + (v_T - v_1)e^{-t/\tau} & t > 0 \end{cases}$

where $v_T = v(0) = 1$ V and $v_1 = 4$ V

$\tau = R_1 C = (10 \times 10^3)\ \Omega \times (40 \times 10^{-6})\ \mu F = 0.40\ s$

$v(t) = \begin{cases} 1 & t = 0^- \\ 4 - 3e^{-t/0.40} & t > 0 \end{cases}$

From (2),
$v_o = -R_f C\frac{dv}{dt} = -(30 \times 10^3)\ \Omega \times (40 \times 10^{-6})\ F \times \left(3 \times \left(\frac{1}{0.40}\right)\right)e^{-t/0.40}$

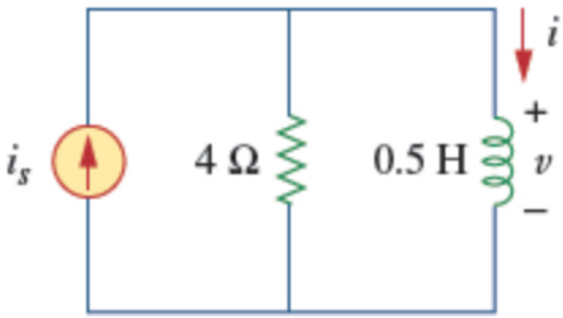
$v_o =$ -9.00 $e^{-t/0.40}$, $t > 0$ or $(-9.00e^{-t/0.40})\,u(t)$ V

The voltage v_o is $((-9.00)e^{-t/0.40})u(t)$ V.

6.

value:
10.00 points

In the given circuit, i_s changes from 6 A to 10 A at $t = 0$, that is, $i_s = [\ 6u(-t) + 10u(t)\ ]$ A. Find $v(t)$ and $i(t)$.

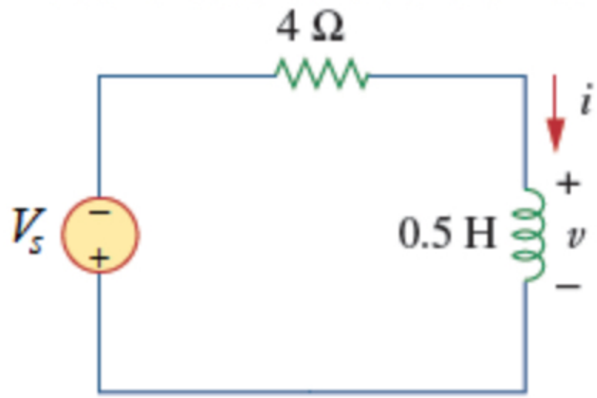


The current $i(t) = ($ 10 ± 2% $) + ($ -4 ± 2% $)e^{-}$ 8 ± 2% t) $u(t)$ A.

The voltage $v(t) = ($ 16 ± 2% $)e^{-}$ 8 ± 2% t $u(t)$ V.

Explanation:

The current source is transformed as shown below.



Here, $V_s = 4i_s$.

$$\tau = \frac{L}{R} = \frac{\left(\frac{1}{2}\right)\text{H}}{4\,\Omega} = \frac{1}{8}\ \text{s}$$

$i(0) = 6$ A

$i(\infty) = 10$ A

$$i(t) = i(\infty) + \left(i(0) - i(\infty)\right)e^{-t/\tau}$$

$$i(t) = \left(10 + (\text{-}4)\,e^{-8t}\right)u(t)\ \text{A}$$

$$v(t) = L\frac{di}{dt} = \left(\frac{1}{2}\ \text{H}\right)\left(\text{-}4\right)\left(-8\right)e^{-8t}$$

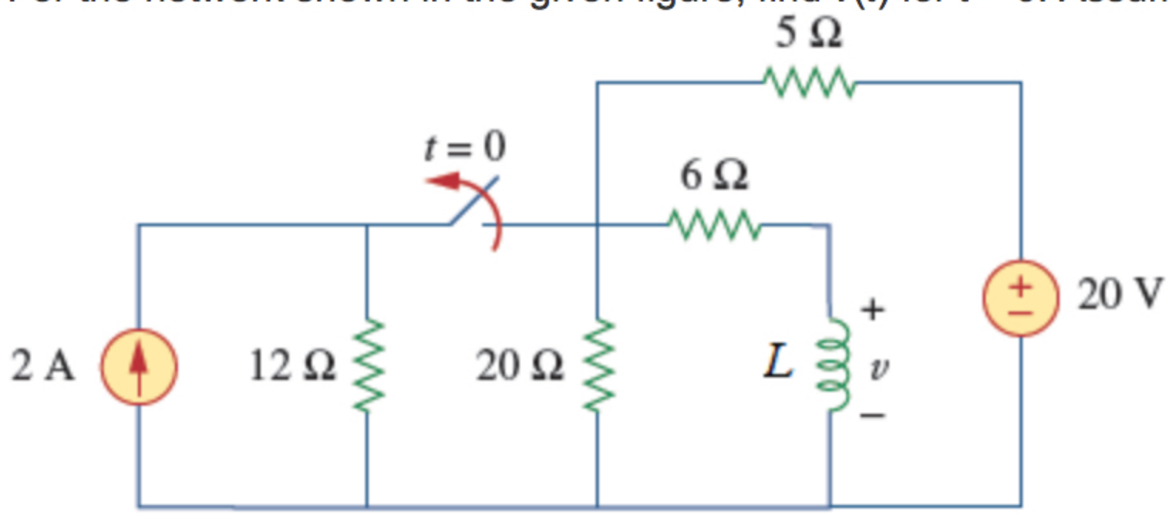
$$v(t) = 16\,e^{-8t}u(t)\ \text{V}$$

The current $i(t) = (10 + (-4)e^{-8t})\ u(t)$ A.
The voltage $v(t) = (16)e^{-8t}\ u(t)$ V.

7.

value:
10.00 points

For the network shown in the given figure, find $v(t)$ for $t > 0$. Assume $L = 1.3$ H.



The voltage $v(t)$ = -4.00 ± 1% $e^{-t/}$ 0.13 ± 1% V for $t > 0$.

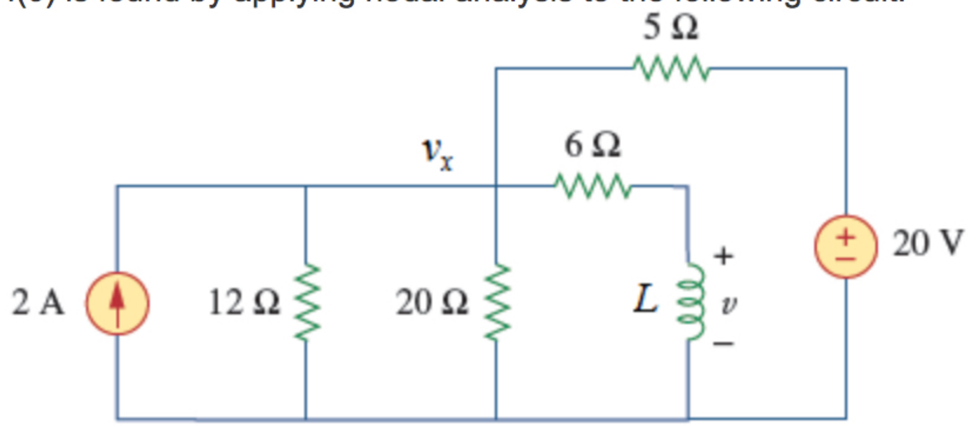
Explanation:

The equivalent resistance is calculated as follows:
$R_{eq} = 6\ \Omega + (20\ \Omega \parallel 5\ \Omega) = 6\ \Omega + 4\ \Omega = 10\ \Omega$

The time constant $\tau = \frac{L}{R_{eq}} = \frac{1.4\ \text{H}}{10\ \Omega} = 0.14\ \text{s}$.

$i(t) = i(\infty) + (i(0) - i(\infty))e^{-t/\tau}$

i(0) is found by applying nodal analysis to the following circuit.



$2\ \text{A} + \frac{20\ \text{V} - v_x}{5\ \Omega} = \frac{v_x}{12\ \Omega} + \frac{v_x}{20\ \Omega} + \frac{v_x}{6\ \Omega} \rightarrow v_x = 12\ \text{V}$

$i(0) = \frac{v_x}{6\ \Omega} = 2\ \text{A}$

Since $20\ \Omega \parallel 5\ \Omega = 4\ \Omega$,

$i(\infty) = \frac{4\ \Omega}{4\ \Omega + 6\ \Omega}(4\ \text{A}) = 1.6\ \text{A}$

$i(t) = 1.6 + (2 - 1.6)\ e^{-t/0.14}\ \text{A} = 1.6 + 0.4e^{-t/0.14}\ \text{A}$

$v(t) = L\frac{di}{dt} = 1.4\ \text{H} \times (0.4)\left(-\frac{1}{0.14}\right)e^{-t/0.14}$

$v(t) = -4.00\ e^{-t/0.14}\ \text{V}$

The voltage $v(t) = -4.00e^{-t/0.14}$ V for $t > 0$.

8.

value:
10.00 points

An RC circuit consists of a series connection of a 160-V source, a switch, a 34-MΩ resistor, and a 15-μF capacitor. The circuit is used in estimating the speed of a horse running a 6-km racetrack. The switch closes when the horse begins and opens when the horse crosses the finish line. Assuming that the capacitor charges to 85.6 V, calculate the speed of the horse.

The speed of the horse is *15.364 ± 3%* m/s.

Explanation:

The speed of horse is calculated as given follows:
$v(\infty) = 170 \text{ V}$

$v(0) = 0$ V
$\tau = RC = (34 \times 10^6)\,\Omega \times (15 \times 10^{-6})\text{ F} = 510 \text{ s}$

$v(t) = v(\infty) + \left(v(0) - v(\infty)\right)e^{-t/\tau} \rightarrow 85.6 = 170\left(1 - e^{-t/510}\right)$

Solving for t gives
$t = 510 \times \ln 2.014 = 357.12 \text{ s}$

$\text{speed} = \frac{7000 \text{ m}}{357.12 \text{ s}} = 19.601 \text{ m/s}$

The speed of the horse is 19.601 m/s.

9.

value:
10.00 points

A capacitor with a value of 70 mF has a leakage resistance of 2 MΩ. How long does it take the voltage across the capacitor to decay to 40% of the initial voltage to which the capacitor is charged? Assume that the capacitor is charged and then set aside by itself.

The voltage across the capacitor takes [35.634 ± 2%] hours to decay to 40% of the initial voltage.

Explanation:

The voltage across a charged capacitor is equal to
$v_C(t) = v_C(0)\ e^{-t/\tau}$

Where $\tau = R_{leak}C = \left(2 \times 10^6\right)\ \Omega \times \left(60 \times 10^{-3}\right)\ \text{F} = 12 \times 10^4\ \text{s}$.

Thus, $0.4v_C(0) = v_C(0)e^{-t/120000}$

$\rightarrow -\frac{t}{120000} = \ln(0.4) = -0.91629$

$\rightarrow t = 109.955\ \text{ks}$

$\rightarrow t = 30.543\ \text{hours}$

The voltage across the capacitor takes 30.543 hours to decay to 40% of the initial voltage.